Optimal Networks, Congestion and Braess' Paradox

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What is an Optimal Network?

We are interested in how to deliver efficiently information in a communications network (e.g. minimising the time to deliver IP packets in the Internet)

Possible Approaches

- Given a network 'find' an algorithm to optimise the delivery of packets

R. Guimerà *et al.*, Optimal network topologies for local search with congestion, Physical Review Letters,**89**, 2002

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- Given a network 'find' an algorithm to optimise the delivery of packets
- Given a packet delivering algorithm 'build' a network that is optimal for this algorithm

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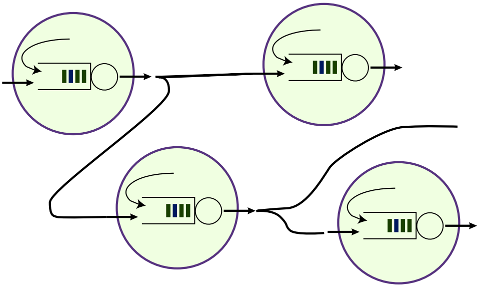
The Network

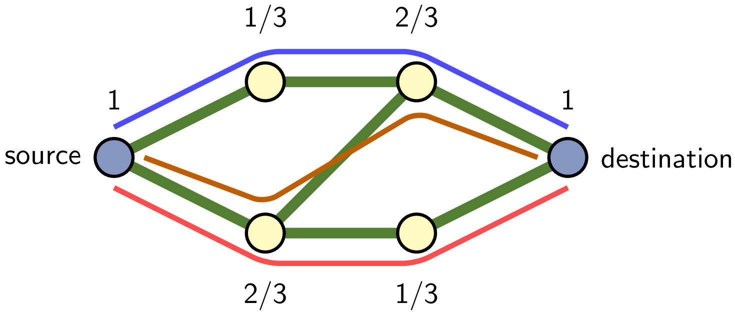
- Fixed number of nodes and links.
- Each node is a source of traffic and has a queue (M/M/1).
- Each node produces the same amount of traffic.
- shortest–path routing



Traffic Load and Centrality

- Estimate the traffic load at node i using the Betweenness Centrality

$$\text{Betweenness} = \mathcal{C}_B = \frac{\text{number of shortest–paths that visit a node}}{\text{number of different shortest paths}}$$



Delay and Congestion

- Assumption: routing using shortest–path.

$$\text{average delay} = \bar{\ell} + 1/(N(N-1)) \sum_{i=1}^{N} \bar{\mathcal{C}}_B(i) \bar{W}_i$$

average length of route $=\bar{\ell}$, number of nodes $=N$ From Little's law

Average time that a packet spends in queue i plus service time

$$\bar{W}_i = \rho_i/(1-\rho_i)(1/\mu), \quad \text{where} \quad \rho = \Lambda/\mu$$

Load–node $\Lambda_i = \lambda N \bar{\ell} \mathcal{C}_B(i)$
Congestion (queuen node m)

$$\boxed{\lambda_c = (\mu(N-1))/\mathcal{C}_B(m)}$$

Zhao *et al.*, Physical Review E, **71**, 2005

Re–wiring the Network – Homogenous Networks

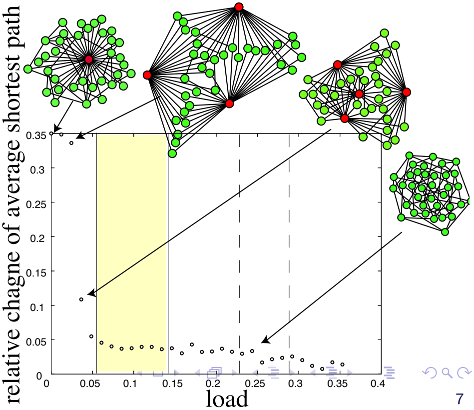
- Given a load λ
- the number of nodes N and links L
- find the network with minimum average delay

  The rewiring is done using simulated annealing

Measure the change of the network structure by looking at the relative change of the shortest path

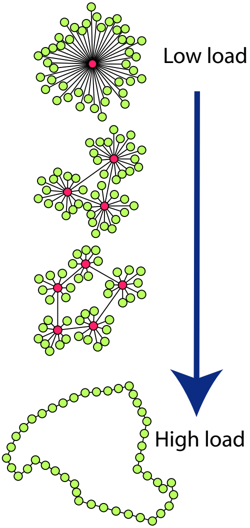
$$\pi' = \frac{\bar{\ell}^*}{\bar{\ell}} - 1$$

where $\bar{\ell}^*$ is the average shortest path of the optimal network closest to congestion.



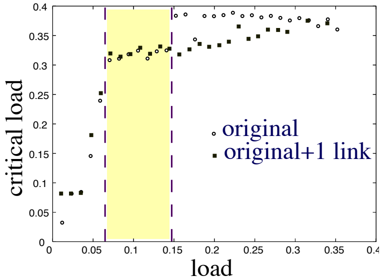
Some Properties

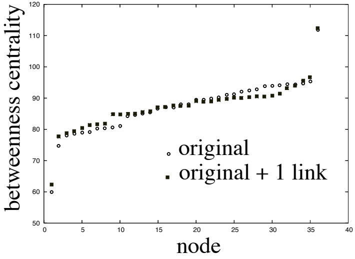
- For simple networks we can evaluate analytically which network is optimal as a function of the load.
- the girth of the network is important when evaluating link failure
- for high load, if the network is regular, it can be classified using concepts of graph theory
- Regular networks are robust (resilient to node, link failures).



Braess' Paradox

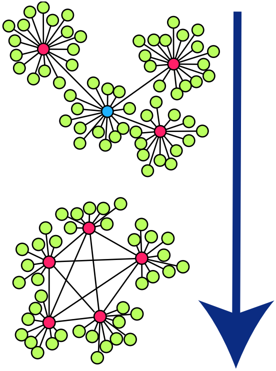
- take an optimal network
- add one link
- optimise the network again
- compare the original network and the network with one extra link





Heterogeneous Networks

- For low loads the network shows a hierarchical structure
- For medium loads the network has a core which can be very well connected (like the AS–Internet graph)
- For high load the network has a regular structure



Extensions and Connection with other Fields of Research

- We have used a different routing algorithm (based on flow–balance) and different queues mechanism (M/D/1), (M/M/k). The results are similar.

Connections with other Fields of Research

- Entangled Networks. Synchronisation of dynamical systems.
- Robust networks (robust to attacks and failures).
- Error correcting codes.

L. Donetti *et al.*, Entangled networks, super–homogeneity and optimal network topology, arXiv:cond-mat/0502230v1, 2006

A.H Dekker and B. D. Colbert, Network Robustness and Graph Topology, 27th Australassian Computer Science

Confernce, 2004

Colouring Graphs.